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Pythagoras says currency hedge

You'd be forgiven for believing that there can't possibly be a connection between Pythagorean geometry and portfolio risk. However, we intend to show that this fundamental and ancient equation has, surprisingly, a very practical relevance to modern-day investors.

This paper will discuss three observations on the well-known portfolio risk equation as it pertains to equity and currency, the two assets that comprise an unhedged international equity portfolio. Specifically, we will demonstrate that:

- Currency exposure can lever a portfolio, and can therefore increase volatility despite a correlation of less than one between currency and equity.
- Re-arranging the portfolio risk equation to solve for correlation, can be a very useful exercise. For given currency and equity volatility forecasts, it can yield an equation that determines the correlation that must exist between currency and equity so that the volatility of a hedged and an unhedged portfolio are equal. This gives investors practical insights into when they should, and should not, hedge their currency risk.
- Under two specific simplifying assumptions, the portfolio risk equation reduces to the Pythagorean equation, and that a simple proof can be invoked to prove that, under these assumptions, retaining currency in a portfolio is always risk additive.

Angle one / currency in a portfolio results in a total asset weight that is greater than one (100%), and is therefore implicitly levered

Many finance professionals have, at some point in their careers, been exposed to the portfolio risk equation for a two asset portfolio:

$$\sigma_P^2 = \sigma_A^2 w_A^2 + \sigma_B^2 w_B^2 + 2\, w_A\, w_B\, cov_{AB} \qquad (1)$$

In words, this states that the risk of a portfolio is a function of three variables: (1) the weighted variance of asset A, (2) the weighted variance of asset B, and (3) the weighted covariances of both assets. However, in our view, practitioners are more likely to discuss asset co-movements in terms of correlation rather than covariance, the former having more intuition than the latter. Therefore, using the relationship $cov_{AB} = \sigma_A\, \sigma_B\, \rho_{AB}$, the portfolio risk equation can also be expressed as:

$$\sigma_P^2 = \sigma_A^2 w_A^2 + \sigma_B^2 w_B^2 + 2\, \sigma_A\, \sigma_B\, w_A\, w_B\, \rho_{AB} \qquad (1)$$

This risk equation is often taught under two (sometimes implicit) assumptions—that the weights of the assets are positive, and that they sum to one (100%). Both of these assumptions are reasonable; they effectively say that no shorting is allowed (w > 0), and that the portfolio is unlevered (w = 1, or 100%)

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In our experience, most professionals rely on these assumptions in order to evaluate portfolio risk. And, in doing so, they are accustomed to believing that any two assets that have a correlation of less than one can often be combined to produce a better outcome than holding either in its entirety (the "free lunch" of portfolio diversification). However, while we agree this is true much of the time for unlevered portfolios, we think that this could be a fallacy when leverage is introduced.

To demonstrate this, consider what happens if the second assumption is relaxed, and the total weight of the assets in the portfolio is allowed to exceed one (100%). We argue that this occurs in international equity investing when one of the "assets" in the portfolio is currency (we put the term in quotes given the question over whether or not currency is in fact an asset). If an investor buys international equity, and doesn't hedge the international currency in which that equity is denominated, then they are as long the currency as they are the equity. In this situation the only weights that now make sense are one (100%) for each asset class, leading to a total weight of two (200%) for the portfolio as a whole. Hence our assertion that currency plays the role of levering a portfolio. Anytime an investor's total notional asset exposure is greater than their investment amount, then leverage of some description has been introduced.

Here's a specific example of what we mean in the case of currency exposure. Any U.S. investor who bought the Eurozone stock market on an unhedged basis would achieve a return of 21%, if both the equities and the euro were to appreciate by 10% over the course of their holding period (1.10 * 1.10). Such a return is compatible only with a weight of one (100%) in the portfolio risk equation to both equity and currency. Note that, strictly speaking, we should also be extending the risk equation to include the variance of the cross product term (the part that takes the return from 20% to 21%), but, for the sake of simplicity, and because the cross-products are on a small scale compared to the equity and currency returns, we choose to ignore them.

To clarify what we have said so far, we will run two distinct examples through the portfolio risk equation. The first example below demonstrates the "traditional" version of the equation where the weights sum to a total of one (100%), and the second example shows the "levered" version using currency, where the weights sum to two (200%). In these examples, and throughout the rest of the paper, we will set the volatilities of the two assets at 16% and 10%. However, these are not purely arbitrary numbers. We actually view them as reasonable estimates of empirical developed market equity risk and empirical developed market currency risk. That said, we don't view our analysis as being contingent on these particular numbers, we are simply choosing something reasonable to facilitate the below discussion.

First, here's the portfolio volatility of a 50%:50% portfolio of domestic equity, assuming $100 is invested in two assets with volatilities of 16% and 10%, and a correlation of zero.

$$\sigma_P^2 = \sigma_A^2\, w_A^2 + \sigma_B^2\, w_B^2 + 2\,\sigma_A\ \sigma_B\, w_A\ w_B\ \rho_{A,B} \qquad (1)$$

$$\sigma_P^2 = 0.16_A^2\, 0.5_A^2 + 0.10_B^2\, 0.5_B^2$$

$$\sigma_P^2 = 0.0064 + 0.0025$$

$$\sigma_P = 9.43\%$$

Second, we show the portfolio volatility of a 100% unhedged international equity portfolio under the assumptions that; $100 is invested in international equity with a volatility of 16%, the international currency has a volatility of 10%, and the correlation between the two is zero.

$$\sigma_P^2 = \sigma_E^2\, w_E^2 + \sigma_{FX}^2\, w_{FX}^2 + 2\,\sigma_E\ \sigma_{FX}\, w_E\ w_{FX}\ \rho_{E,FX} \qquad (1)$$

$$\sigma_P^2 = 0.16_A^2 + 0.10_B^2$$

$$\sigma_P^2 = 0.0256 + 0.0100$$

$$\sigma_P = 18.87\%$$

The first outcome is the standard one that we would expect, that the effect of equally combining two uncorrelated assets is that the risk of the portfolio is lower than the standalone risk of either asset. This is the well-known benefit to diversification. The second outcome though may come as a surprise to some. Despite combining two uncorrelated assets, the portfolio has become riskier.

The volatility of 18.87% is higher than the standalone volatilities of both of the individual components. It seems as though the additional risk from the currency leverage more than offsets the risk reduction advantage of the assets being uncorrelated.

The reason that we find this result important is because it runs counter to what we think may be a behavioral default that investors, quite reasonably, have—believing that uncorrelated assets are almost always effective risk diversifiers within a portfolio. Again, we agree with this in unlevered portfolios, but when the new asset is also creating implicit portfolio leverage, we need to adjust our assumptions.

Angle two / the correlation "break-even" equation

Another very informative use of the levered risk equation is to pose, and answer, the following question: what correlation needs to exist between equity and currency, such that the risk of the unhedged portfolio, and the hedged portfolio are equal? This number can then be compared to the investor's own correlation assumption to see if it seems reasonable.

We took the following route to get to this equation.

Firstly, we started with the standard risk equation (Equation (1) from above):

$$\sigma_P^2 = \sigma_E^2\, w_E^2 + \sigma_{FX}^2\, w_{FX}^2 + 2\,\sigma_E\ \sigma_{FX}\, w_E\ w_{FX}\ \rho_{E,FX} \qquad (1)$$

Next, we applied the requirement that was discussed earlier in the paper, that the weight in each asset was one (100%), and then we set the volatility of the unhedged portfolio (σ_P^2) equal to the volatility of equity (which can effectively be thought of as the volatility of a fully hedged portfolio (σ_E^2)).

This gives:

$$\sigma_P^2 = \sigma_E^2 + \sigma_{FX}^2 + 2\,\sigma_E\ \sigma_{FX}\,\rho_{E,FX}$$

Deducting σ_P^2 and σ_E^2 from each side (allowable because we have equated them):

$$\sigma_{FX}^2 + 2\,\sigma_E\ \sigma_{FX}\,\rho_{E,FX} = 0$$

Dividing by $2\,\sigma_E\ \sigma_{FX}$:

$$\sigma_{FX}^2/2\,\sigma_E\ \sigma_{FX} + \rho_{E,FX} = 0$$

Simplifying, and rearranging:

$$\rho_{E,FX} = -\frac{1}{2}\sigma_{FX}/\sigma_E \qquad (2)$$

We think Equation (2) is important because it enables an investor, armed with an assumption about the risk of international equity and currency, to gauge, in a relatively straightforward way, their "correlation break-even"—effectively the level of correlation that would have to persist so that investors are indifferent to being hedged, or not, from a risk perspective.

For example, suppose an investor had the assumptions used above, that international equity had a volatility of 16%, and the currency had a volatility of 10%. The equation would conclude that, to be indifferent to hedging (or, said another way, for the risk of the hedged and unhedged portfolios to equate) the correlation between equity and currency would have to be –0.31. If the correlation were lower than that the currency exposure would reduce the risk in an unhedged equity portfolio, and, if it were higher, then the currency exposure would add to risk.

Note that the equation is not perfect. If an investor believed that currency was going to be more than twice as risky as equity then it would give a correlation number less than negative one (which is not possible, and would effectively have to be read as "hedge"). But the presence of the minus sign on the right hand side is instructive. It tells us that any assumptions for currency risk that are greater than zero (which must be all assumptions) result in required correlations being negative.

Exhibit one shows the result that the equation would give for a number of assumptions for equity and currency volatility. It emphasizes the need for a negative correlation regardless of volatility assumptions, as well as two other intuitive conclusions:

_ As equity volatility increases, currency necessarily becomes a smaller component of overall risk, and there is a lower burden to its inclusion (for example, for a currency volatility of 10%, an investor needs to believe that a correlation of –0.50 (or lower) exists if equity volatility is also 10%. However, if equity volatility increases to 20%, they only need to see –0.25 (or lower).

_ As currency volatility increases, it necessarily becomes a larger component of overall risk, and there is a higher burden to leaving currency exposure unhedged (for example, for an equity volatility of 16%, an investor only needs to believe that a correlation of –0.31 (or lower) exists if currency volatility is 10%. However, if currency volatility increases to 12%, they now need to see a correlation of –0.38 (or lower).

Angle three / from portfolio risk to pythagoras

Our final observation is what we considered to be a rather elegant proof that, under the assumption of zero correlation, the risk of an unhedged portfolio is always greater than that of a hedged portfolio. We often hear from investors that they view developed market currencies as approximately uncorrelated with equity markets (a view that we would agree with) and that therefore it should act as a diversifier in their portfolio (a view that we would agree with for a traditional "sum to one (100%)" portfolio, but which we would disagree with when the leverage of currency is introduced). We hope they'll appreciate our rebutting this argument by invoking the Pythagorean equation.

EXHIBIT ONE: A TABLE OF "BREAK-EVEN" CORRELATIONS THAT AN INVESTOR MUST EXPECT TO BE INDIFFERENT BETWEEN HEDGING AND NOT HEDGING CURRENCY EXPOSURE, FROM A RISK PERSPECTIVE.

EQUITY VOLATILITY \ CURRENCY VOLATILITY	4%	5%	6%	7%	8%	9%	10%	11%	12%
6%	–0.33	–0.42	–0.5	–0.58	–0.67	–0.75	–0.83	–0.92	–1.00
7%	–0.29	–0.36	–0.43	–0.50	–0.57	–0.64	–0.71	–0.79	–0.86
8%	–0.25	–0.31	–0.38	–0.44	–0.50	–0.56	–0.63	–0.69	–0.75
9%	–0.22	–0.28	–0.33	–0.39	–0.44	–0.50	–0.56	–0.61	–0.67
10%	–0.20	–0.25	–0.30	–0.35	–0.40	–0.45	–0.50	–0.55	–0.60
11%	–0.18	–0.23	–0.27	–0.32	–0.36	–0.41	–0.45	–0.50	–0.55
12%	–0.17	–0.21	–0.25	–0.29	–0.33	–0.38	–0.42	–0.46	–0.50
13%	–0.15	–0.19	–0.23	–0.27	–0.31	–0.35	–0.38	–0.42	–0.46
14%	–0.14	–0.18	–0.21	–0.25	–0.29	–0.32	–0.36	–0.39	–0.43
15%	–0.13	–0.17	–0.20	–0.23	–0.27	–0.30	–0.33	–0.37	–0.40
16%	–0.13	–0.16	–0.19	–0.22	–0.25	–0.28	–0.31	–0.34	–0.38
17%	–0.12	–0.15	–0.18	–0.21	–0.24	–0.26	–0.29	–0.32	–0.35
18%	–0.11	–0.14	–0.17	–0.19	–0.22	–0.25	–0.28	–0.31	–0.33
19%	–0.11	–0.13	–0.16	–0.18	–0.21	–0.24	–0.26	–0.29	–0.32
20%	–0.10	–0.13	–0.15	–0.18	–0.20	–0.23	–0.25	–0.28	–0.30
21%	–0.10	–0.12	–0.14	–0.17	–0.19	–0.21	–0.24	–0.26	–0.29
22%	–0.09	–0.11	–0.14	–0.16	–0.18	–0.20	–0.23	–0.25	–0.27
23%	–0.09	–0.11	–0.13	–0.15	–0.17	–0.20	–0.22	–0.24	–0.26
24%	–0.08	–0.10	–0.13	–0.15	–0.17	–0.19	–0.21	–0.23	–0.25
25%	–0.08	–0.10	–0.12	–0.14	–0.16	–0.18	–0.20	–0.22	–0.24

Hypothetical illustration. Not representative of any particular security or product.

As before, we start with the normal risk equation:

$$\sigma_P^2 = \sigma_A^2\, w_A^2 + \sigma_B^2\, w_B^2 + 2\, \sigma_A\ \sigma_B\, w_A\ w_B\ \rho_{A,B} \qquad (1)$$

Setting the weights of A and B to one (100%):

$$\sigma_P^2 = \sigma_A^2 + \sigma_B^2 + 2\, \sigma_A\ \sigma_B\ \rho_{A,B}$$

Assuming the correlation between the assets is equal to zero:

$$\sigma_P^2 = \sigma_A^2 + \sigma_B^2$$

This final, simplified equation, is of course the famous Pythagorean equation for the hypotenuse of a right-angled triangle:

$$c^2 = a^2 + b^2$$

Applied to the risk of the portfolio, it says in words: the variance of an unhedged portfolio is equal to the variance of equity risk (hedged portfolio), plus the variance of currency risk, under the assumption of zero correlation, and under the requirement that the weights sum to two (200%).

Since, for positive values of the Pythagoras equation, it has to be true that:

$a^2 + b^2 > a^2$ and $a^2 + b^2 > b^2$

And since, from above, c^2 is equal to $a^2 + b^2$, it follows that:

$c^2 > a^2$ and $c^2 > b^2$

Rewriting this in financial notation:

$\sigma_P^2 > \sigma_A^2$ and $\sigma_P^2 > \sigma_B^2$

In words, this says that, for a correlation of zero, and under the requirement that the asset weights sum to two (200%), the volatility of an unhedged portfolio is always greater than the volatility of a hedged portfolio (and always greater than the volatility of the currency). So what Pythagoras is telling us is that, if we genuinely believe that currency and equity returns are uncorrelated, then leaving currency in the portfolio can only add to risk.

Frankly, we find this direct link between Pythagorean geometry and the risk of a portfolio of equity and currency to be extraordinary. And, though the similarities between portfolio risk and the cosine rule are already known, we have not before seen this direct derivation from the risk equation to the Pythagorean equation.

Furthermore, we do not believe that this is simply of academic interest. Indeed, the key point for us is the practical lesson that this derivation results in—that investors who believe that currency and equity returns are uncorrelated, and therefore are content to leave their currency risk unhedged in the belief that it is reducing risk, are demonstrably wrong.

Conclusions

A close look at the portfolio risk equation as it applies to a portfolio of equity and currency (an international investment that is currency unhedged) results in three conclusions:

First, that the only weight that makes sense for each asset class (assuming the portfolio is fully invested) is one (100%) for the equity, and one (100%) for the currency. As such, the total portfolio has a weight of two (200%) and acts like a levered portfolio. Contrary to normal uses of the risk equation, this can result in potential overall portfolio volatility that is higher than even the standalone volatility of the riskier of the two assets (an impossibility in the standard, unlevered application).

Second, that setting the risks of the unhedged and hedged portfolio equal, and rearranging, results in a very simple equation that shows the "correlation break-even" that an investor must need in order to leave their currency unhedged (from a risk perspective). Indeed the equation states that values specifically more negative than the result will be needed in practice if retaining currency exposure is to reduce overall risk.

Finally, that under the reasonable assumption of zero correlation, the levered portfolio risk equation reduces to the Pythagorean equation. A simple proof shows us that, just as the hypotenuse of a right-angled triangle must be longer than either the opposite or adjacent sides, so, by direct analogy, the volatility of an unhedged portfolio must be greater than that of a hedged portfolio.

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